

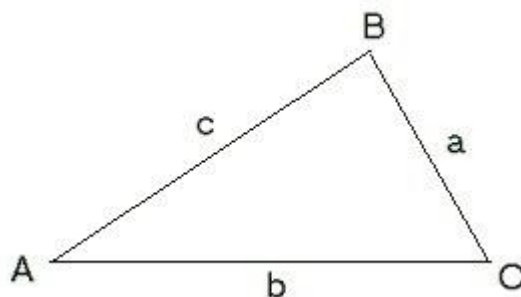
# The Laws of Sine and Cosine

## Objectives:

- Given a triangle and three quantities (ASA, SAS, SSS, SSA, AAS) of data about the triangle, use the law of sines, or the law of cosines to determine the three remaining unknowns.

## Discussion

Every triangle has three vertices and three sides. In the picture to the right the three vertices are A, B, and C, and the sides opposite these vertices are a, b, and c, respectively. The angle at each vertex is given the same name as the vertex. Therefore, the three angles are also named A, B, and C.



The **Law of Cosines** states that:

$$a^2 = b^2 + c^2 - 2bc \cos(A)$$

$$b^2 = a^2 + c^2 - 2ac \cos(B)$$

$$c^2 = a^2 + b^2 - 2ab \cos(C)$$

Use the law of cosines when you are given SAS, or SSS, quantities. For example: If you were given the lengths of sides b and c, and the measure of angle A, this would be SAS. SSS is when we know the lengths of the three sides a, b, and c.

The **Law of Sines** states that

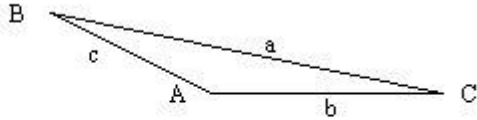
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Use the law of sines when you are given ASA, SSA, or AAS. An example of ASA is when you are given the measure of angles A, and C, and the length of side b. An example of SSA is when you are given the sides c, and a, and angle C. An example of AAS is when you are given angles C and A, and side c.

# Examples

## Example 1



Angle B = 14.967 degrees side a = 74.185  
side c = 32.945. Find angles A, C, and side b.

Notice that side a, angle B, and side c are given.  
This is SAS. We will need to begin the problem  
with the **law of cosines**.

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$b = \sqrt{a^2 + c^2 - 2ac \cos B}$$

$$a = 74.185, c = 32.945,$$

$$B = 14.967 \text{ degrees}$$

$$b = 43.204$$

Now lets find angle C. Since we now know angle  
B, and sides b, and c, we will use the **law of sines**.

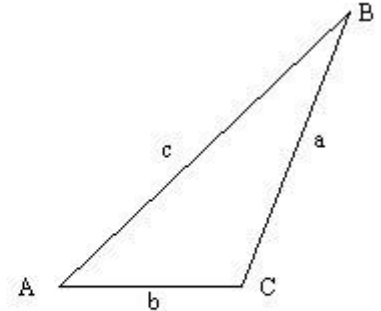
$$\frac{\sin C}{c} = \frac{\sin B}{b}$$

$$C = \sin^{-1}\left(\frac{c \sin B}{b}\right)$$

$$C = 11.358 \text{ degrees}$$

Since the sum of the three angles is 180, then A =  
 $180 - B - C = 153.675$  degrees.

## Example 2



Angle A = 43.202 degrees angle C = 111.622 degrees,  
side a = 79.401. Find angle B, and sides b and c.

Notice that angles A and C are given, and side a.  
This is AAS. Lets begin the problem by finding  
angle B. Since  $B = 180 - A - C$ , and A = 43.202  
degrees, and C = 111.622 degrees, then B = 25.176  
degrees.

Now lets find side b. By the **law of sines**:

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$

$$b = \frac{a \sin B}{\sin A}$$

$$b = 49.341$$

Using the law of sines again to find side c:

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$

$$c = \frac{a \sin C}{\sin A}$$

$$c = 107.825$$